

## **A POSITION ESTIMATION METHOD BASED ON MODIFIED INTEGRATOR OF PMSM SENSORLESS CONTROL FOR HIGH-SPEED VEHICLES**

Qian Yuan, Zhongping Yang, Ling Cui, Taiyuan Hu, and Fei Lin  
School of Electrical Engineering,  
Beijing Jiaotong University, 100044, Beijing, China

### **ABSTRACT**

In the high-speed traction permanent magnet synchronous motor (PMSM) drive system, control methods without position and speed sensors are adopted because of occupying smaller space, lower cost, higher reliability and less sensitivity to the environment which is more suitable for motors using in traction vehicles. According to the stator voltage and measured current, the stator flux is integrating from the difference value of the voltage, where back-EMF can be calculated and rotor position can be estimated. Based on the estimating method of integrating back-EMF, this paper proposes a modified integrator to solve the problems produced by the pure integrator, which brings up DC offset to estimating motor flux. Simulation and experiment results show that this integrator can accurately estimate both the magnitude and the phase angle of the motor flux, including the speed of PMSM. Besides, the performance of the integrator shows that it has good dynamic and steady characteristics.

### **KEYWORDS**

High-speed train, PMSM, flux estimation, integrator, sensorless control.

### **INTRODUCTION**

Permanent magnet synchronous motors (PMSM) are widely used in more and more fields because of their high efficiency and high power density. In most drive systems, we often use a position sensor such as a shaft sensor or a resolver to detect the signals that contain the angular information. In high-speed traction drive system, however, control methods without position and speed sensors are adopted because they are of higher reliability and less sensitive to environment, moreover, making the motor smaller size and less cost. (Yilmaz Sozer, 1999)

Sensorless PMSM control strategy are mainly divided into three types: One is the open-loop estimating methods based on the electromagnetic relation of motor model, which relays on the accuracy of motor parameters. Another is closed-loop estimating methods based on the observers which are now popular in many fields which need higher speed precision and system reliability, but it is more complex in algorithm which lead to slow respond. The third one is the estimating methods based on the nonideal characters, which tracking the saliency of PMSM and mainly dealing with the low speed estimation. It doesn't rely on the motor model or the parameters, so it has good robustness. But it relays on high frequency signal rejection, so it may bring high frequency noise. (P. Hutterer et al., 2009)

There are, in general, two methods for the first estimation strategy: Estimation based on measured current and estimation based on measured voltage (Jun Hu et al., 1998). In the current-based method, the measured current and motor parameters are required for the motor flux calculation, so it's sensitive to the parameters change during operation. In the voltage-based method, the motor flux can be obtained by integrating the back electromotive force (back-EMF). The parameters only required are the motor resistance and inductance, and in IPMSM control, the angle of the magnet pole is needed as well. It seems that it is much more easier to use the voltage-method than the current-method. However, the implementation of an integrator for flux estimation is no easy task. A pure integrator would bring about DC drift and DC offset problems. A low pass filter can solve these problems, but it may also produce errors in magnitude and phase angle. This paper describes a modified integrating method for flux estimation based on the motor voltage model, which can solve the problems using a pure integrator or a low-pass filter. The performance is studied and verified through simulation. (Budden, A.S. et al., 2005)

### **PMSM MATHEMATIC MODEL AND VECTOR CONTROL STRATEGES**

### ***PMSM mathematic model***

The stator voltage equation in the rotating dq two phase rotor reference frame can be expressed as follow: (Yilmaz Sozer, 1999)

$$\begin{aligned} u_q &= R_s i_q + p\psi_q + \omega_r \psi_d \\ u_d &= R_s i_d + p\psi_d - \omega_r \psi_q \end{aligned} \quad (1)$$

The flux linkage in dq reference frame is

$$\begin{aligned} \psi_q &= L_q i_q \\ \psi_d &= L_d i_d + \psi_f \end{aligned} \quad (2)$$

The torque equation can be expressed as

$$T_e = \frac{3}{2} p \left[ \psi_f i_q + (L_d - L_q) i_d i_q \right] \quad (3)$$

where,

$u_d, u_q$  : d-axis and q-axis stator voltage;

$i_d, i_q$  : d-axis and q-axis stator current ;

$\psi_d, \psi_q$  : d-axis and q-axis flux;

$L_d=L_q$  : d-axis and q-axis inductance;

$\omega_r$  : rotor electrical angular velocity;

$\psi_f$  : rotor magnet flux;

$R_s$  : stator resistance ;

$p$  : differential operator.

### ***PMSM vector control strategies***

According to the differences of permanent magnet position, the PMSM are mainly divided into two types: the surface permanent magnet synchronous motor (SPMSM) and the interior permanent magnet synchronous motor (IPMSM). In SPMSM, the permanent magnet is on the surface of the rotor, and the d-axis inductance and q-axis inductance is the same ( $L_d=L_q$ ). While in IPMSM, where the permanent magnet is inserted into the rotor, the d-axis inductance is usually smaller than the q-axis inductance ( $L_d<L_q$ ). (Xu Junfeng et al., 2004)

#### (1) zero d-axis current control

zero d-axis current control strategy is a widely used vector control strategy in PMSM control. When  $i_d=0$ , the torque equation of the PMSM would be expressed below:

$$T_e = \frac{3}{2} p \left[ \psi_f i_q + (L_d - L_q) i_d i_q \right] = \frac{3}{2} p \psi_f i_q \quad (4)$$

We can conclude from the equation above that when we adopt the zero d-axis current control strategy, the torque of PMSM only relies on q-axis current. And this strategy is more often used in the SPMSM control.

#### (2) Maximum torque current control

As in the IPMSM, the  $L_d \neq L_q$ , when controlling the IPMSM, we usually adopt a control strategy called maximum torque current control, which would get the maximum torque with the minimum current.

#### (3) Field weakening control

When the motor reaches the limit voltage of the inverter,  $i_d$  would be negative to decrease the flux at this time. And we have to increase  $i_d$  and decrease  $i_q$  to keep the motor voltage and inverter voltage balanced. And as a consequence, make the motor speed up.

In this paper, the zero d-axis current control strategy is adopted to control the IPMSM.  $i_d=0$ , the torque equation of the PMSM would be equation (1). The vector diagram in zero d-axis current control is shown in

Figure 1, where  $\vec{i}_s$  is the stator current vector,  $\vec{\psi}_f$  is the rotor flux vector,  $\vec{\psi}_s$  is the stator flux vector. The stator flux equation can be expressed below:

$$\vec{\psi}_s = L_s(\theta_r)\vec{i}_s + \vec{\psi}_f \quad (5)$$

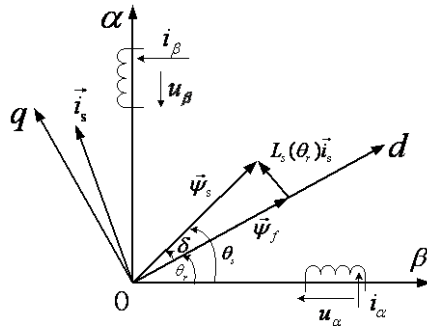


Figure 1 Vector diagram of PMSM in zero d-axis current control

Figure 2 is the diagram of zero d-axis current control system with rotor flux position estimation .

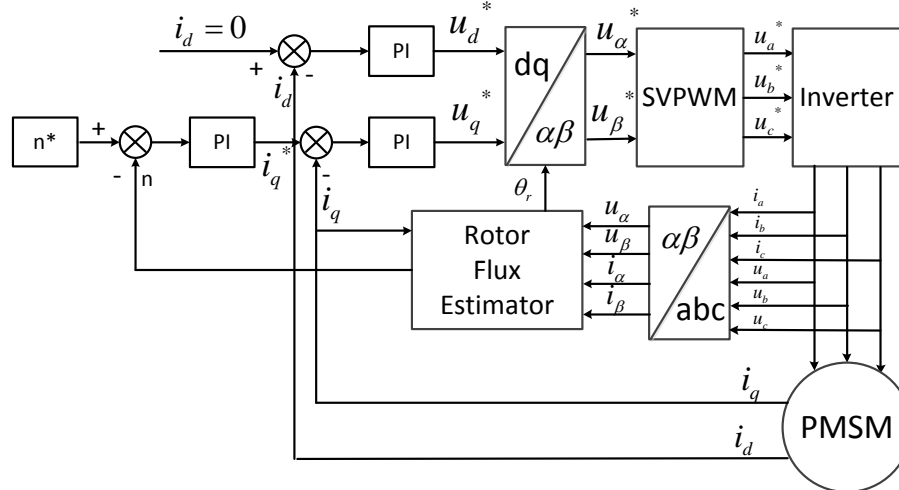


Figure 2 Diagram of zero d-axis current control system with rotor flux position estimation

### PMSM ROTOR FLUX ESTIMATION (Cirincione M et al., 2004)

Based on the PMSM voltage vector equation  $u_s = Ri_s + \frac{d\psi_s}{dt}$ , the stator flux can be expressed as

$$\vec{\psi}_s = \int (\vec{u}_s - R_s \vec{i}_s) dt \quad (6)$$

Transforming into the static two-phase  $\alpha\beta$  reference frame, it is

$$\psi_\alpha = \int (u_\alpha - R_s i_\alpha) dt \quad (7)$$

$$\psi_\beta = \int (u_\beta - R_s i_\beta) dt$$

where,

$\psi_\alpha, \psi_\beta$  :  $\alpha$ -axis and  $\beta$ -axis stator flux.  $u_\alpha, u_\beta$  :  $\alpha$ -axis and  $\beta$ -axis stator voltage.  $i_\alpha, i_\beta$  :  $\alpha$ -axis and  $\beta$ -axis stator current .

According to equation (5), the stator flux in static  $\alpha\beta$  reference frame can be expressed as follow,

$$\begin{pmatrix} \psi_\alpha \\ \psi_\beta \end{pmatrix} = L_s(\theta_r) \begin{pmatrix} i_\alpha \\ i_\beta \end{pmatrix} + \begin{pmatrix} \psi_f \cos \theta_r \\ \psi_f \sin \theta_r \end{pmatrix} \quad (8)$$

The inductance in static  $\alpha\beta$  reference frame can be expressed

$$\begin{aligned} L_s(\theta_r) &= \begin{pmatrix} L_\alpha & L_{\alpha\beta} \\ L_{\beta\alpha} & L_\beta \end{pmatrix} = \begin{pmatrix} \cos \theta_r & -\sin \theta_r \\ \sin \theta_r & \cos \theta_r \end{pmatrix} \begin{pmatrix} L_d & 0 \\ 0 & L_q \end{pmatrix} \begin{pmatrix} \cos \theta_r & \sin \theta_r \\ -\sin \theta_r & \cos \theta_r \end{pmatrix} \\ &= \begin{pmatrix} L_0 + \Delta L \cos 2\theta_r & \Delta L \sin 2\theta_r \\ \Delta L \sin 2\theta_r & L_0 - \Delta L \cos 2\theta_r \end{pmatrix} \end{aligned} \quad (9)$$

where  $L_0 = \frac{1}{2}(L_d + L_q)$ ,  $\Delta L = \frac{1}{2}(L_d - L_q)$ .

So the rotor position angle  $\theta_r$  can be expressed as follow,

$$\begin{aligned} \hat{\theta}_r &= \arctan \left( \frac{\psi_\beta - L_q(\theta_r)i_\beta}{\psi_\alpha - L_d(\theta_r)i_\alpha} \right) \\ &= \arctan \left( \frac{\int (u_\beta - R_s i_\beta) dt - L_q(\theta_r)i_\beta}{\int (u_\alpha - R_s i_\alpha) dt - L_d(\theta_r)i_\alpha} \right) \end{aligned} \quad (10)$$

Obviously, we can easily obtain the stator flux by integrating the back-EMF, which is only related to motor resistance. This method seems to be very simple and respond fast, but the integrating algorithm would cause some problems. When using a pure integrator, if the input contains DC component, the integrator would drift the output into saturation. Besides, when a sine signal is applied to the integrator, a cosine wave is expected at the output, but the cosine wave would bring a DC component at the time of 0, resulting in that the output is cosine wave with a DC bias. Usually we employ a first-order low-pass filter (LPF) to replace the pure integrator to solve the DC offset and the DC drift problems, but it may bring other problems since it would produce errors in both magnitude and phase angle, especially under low speed condition. (Shin-Myung Jung et al., 2008)

### MODIFIED INTEGRATOR FOR PMSM

To solve these problems produced by pure integrator and LPF, we proposed a modified integrator. The output of modified integrator can be expressed as (Jun Hu et al., 1998)

$$y = \frac{1}{s + \omega_c} x + \frac{\omega_c}{s + \omega_c} z \quad (11)$$

where  $x$  is the input of the integrator and  $z$  is the compensation component.

When the input  $x$  passes through the LPF part, assuming the output is  $y_1$ , changes would take place in both magnitude and phase angle of  $y_1$ . So we need to compensate the errors. While  $y$  does not reach the limit  $L$ , the modified integrator performs as a pure integrator where  $y = 1/x$ . While  $y$  reaches the limit of saturation, then  $z = L$  and the output of the modified integrator is

$$y = \frac{1}{s + \omega_c} x + \frac{\omega_c}{s + \omega_c} L \quad (12)$$

where  $L$  is the saturation of block.

Assuming the input  $x$  contains a small DC error  $x_{dc}$ , the output should be [3]

$$y = \frac{1}{s + \omega_c} (x + x_{dc}) + \frac{\omega_c}{s + \omega_c} L = \frac{1}{s + \omega_c} (x + x_{dc} + \omega_c L) \quad (13)$$

The limiting level should be set equal to the expected output magnitude. Assuming  $L = x/s$ , so the equation (12) can be expressed as

$$y = \frac{1}{s + \omega_c} (x + x_{dc} + \omega_c L) = \frac{1}{s + \omega_c} (x + x_{dc} + \omega_c \frac{x}{s}) = \frac{1}{s + \omega_c} x_{dc} + \frac{x}{s} = \frac{1}{\omega_c} x_{dc} + \frac{x}{s} \quad (14)$$

Which can infer that the modified integrator would not be driven into saturation when the  $L$  is properly set.

Figure 3 is a modified integrator with saturable feedback based on coordinate transmission used in the PMSM.

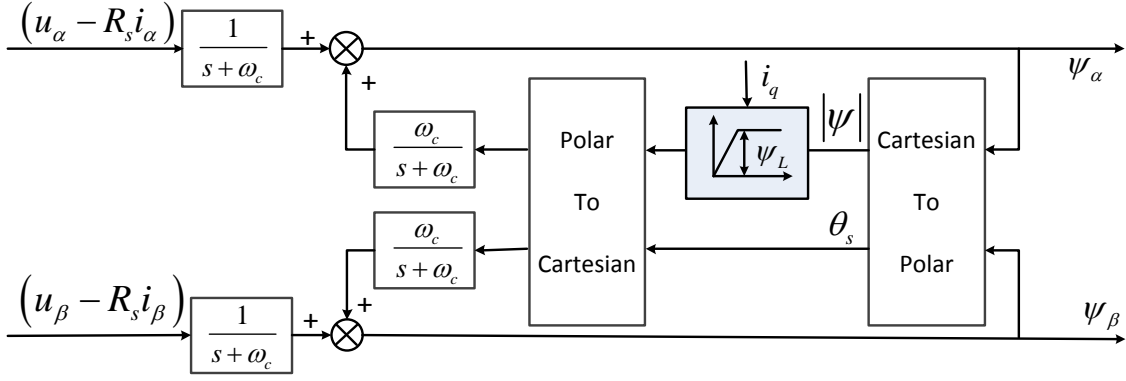


Figure 3 A modified integrator with saturable feedback based on coordinate transmission

## SIMULATION AND RESULT

In the simulation, we use last estimated  $\theta_{r(k-1)}$  instead of  $\theta_{r(k)}$  to calculate the motor inductance. So we can obtain the rotor position as equation (15) shows, and the block diagram is shown in Figure 4.

$$\begin{aligned}\hat{\theta}_{r(k)} &= \arctan\left(\frac{\psi_{\beta} - L_q(\theta_{r(k-1)})i_{\beta}}{\psi_{\alpha} - L_d(\theta_{r(k-1)})i_{\alpha}}\right) \\ &= \arctan\left(\frac{\int(u_{\beta} - R_s i_{\beta})dt - L_q(\theta_{r(k-1)})i_{\beta}}{\int(u_{\alpha} - R_s i_{\alpha})dt - L_d(\theta_{r(k-1)})i_{\alpha}}\right)\end{aligned}\quad (15)$$

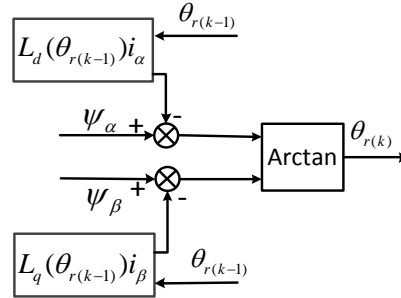
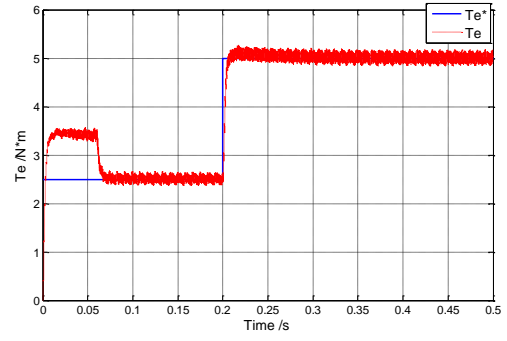
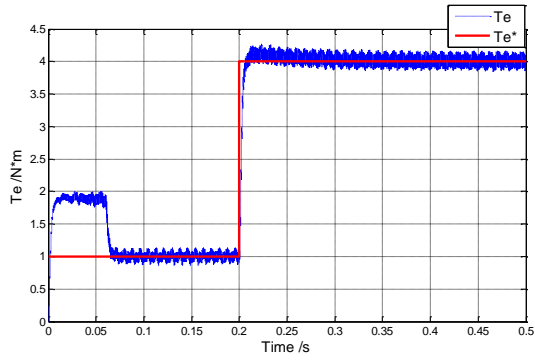


Figure 4 Diagram of rotor flux angle estimation

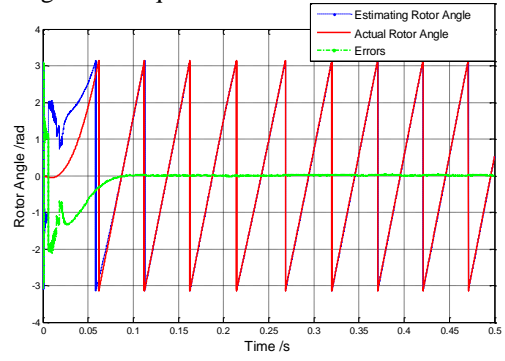
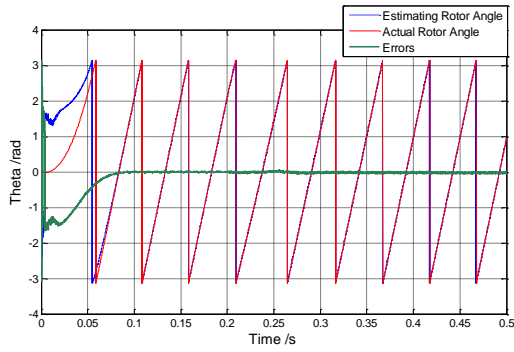
The performance of the proposed integrator is investigated by Matlab/Simulink. The motor used in simulation is a 4-pole IPMSM with whose flux linkage is 0.175Wb,  $L_d$  is 5mH,  $L_q$  is 8.5 mH. The given speed is 600rpm, so the  $\omega$  in steady state is 125 rad/s. The cutoff frequency ( $\omega_c$ ) used in the LP filter and proposed algorithm is 251 rad/s. The motor starts at the speed of 0. During the start process, we use the actual rotor angle for coordinate transformation, and then switch to the estimating rotor angle at 0.18s. The given torque changes at 0.2s. The start process lasts for 0.08s and then reaches the steady state.

The result is shown in Figure 5. On the left side of (a)-(d) is the results of load torque change from 1N\*m to 4 N\*m at 0.2s, while on the right side is the results of load torque change from 2.5 N\*m to 5N\*m. We can see from the figure that the motor go after the given speed at 0.08s and then as the load torque suddenly increasing, the motor speed drops at 0.2s and then slowly goes back to the given speed. During this period the difference between the estimation and actual value is kept within 0.02rad all the time ( $<2^\circ$ ).

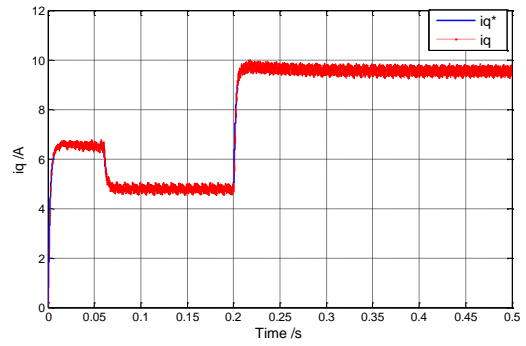
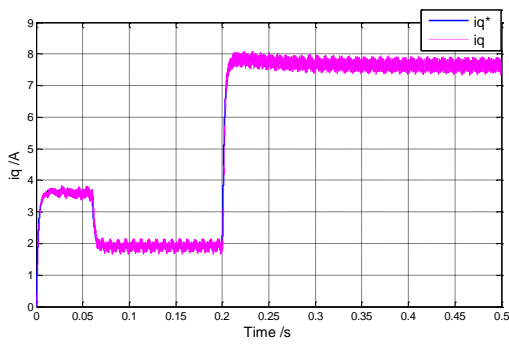
The Figure5(e) is comparing with the errors of motor with different initial rotor position angle. As is shown that different initial rotor angle had little influence to the estimating errors both in regular time and steady-state error.



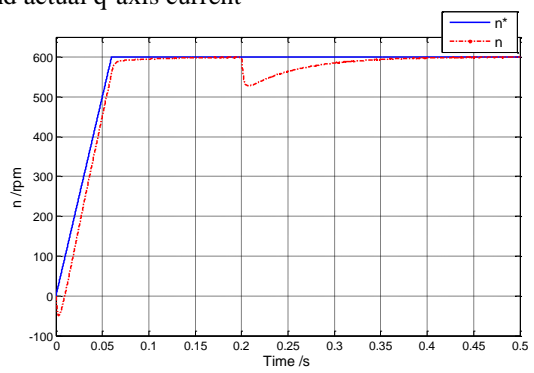
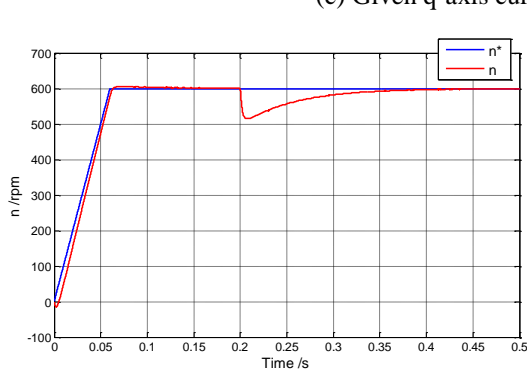
(a) Electromagnetic Torque



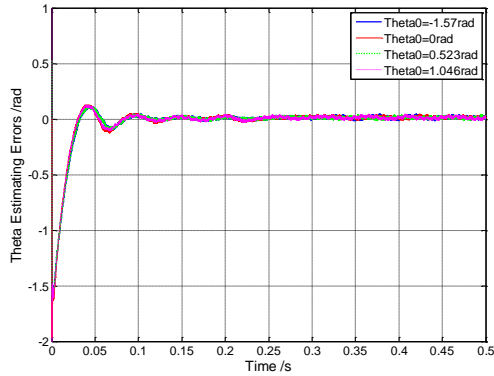
(b) Estimated rotor angle, actual rotor angle and the error



(c) Given q-axis current and actual q-axis current



(d) Given speed and motor speed



(e) Theta Estimating Error of different initial rotor angle

Figure 5 Results of Estimation Simulation

## CONCLUSIONS

The modified integrator proposed in this paper is developed to solve the problems produced by the pure integrator and low-pass filter for rotor flux estimation. The integrator is suitable for motor operation over a wide speed range. Especially during load change operation,  $i_q$  changes along with load change, the error is controlled within 0.02rad. And cases of different initial rotor angle are discussed. The performance of the proposed integrator is verified through simulation.

## ACKNOWLEDGMENTS

The authors gratefully acknowledge the financial support provided by a grant from National Key Technology Support Program (2009BAG12A01-F15-1) .

## REFERENCES

- Yilmaz Sozer (1999). "Direct Adaptive Control of Permanent Magnet Motors", . *Thesis for the Master of Science Degree, Graduate Faculty of Rensselaer Polytechnic Institut*,100-128.
- Jun Hu and Bin Wu (1998). "New Integration Algorithms for Estimating Motor Flux over a Wide Speed Range", *IEEE TRANSACTIONS ON POWER ELECTRONICS*, 13(5), 969-977.
- P. Hutterer, H. Grabner, S. Silber, W. Amrhein and W. Schaefer (2009), "A Study on Systematic Errors Concerning Rotor Position Estimation of PMSM Based on Back EMF Voltage Observation", *Electric Machines and Drives Conference, 2009. IEMDC '09. IEEE International*, 1393-1400.
- Budden, A.S., Holliday, D., Mellor, P.H.(2005). "Zero Speed Sensorless Position Detection for Permanent Magnet Synchronous Machines", *Power Electronics Specialists Conference, IEEE 36th*, 5, 2436-2441.
- Xu Junfeng,Xu Yinglei,Xu Jianping,etal(2004). "A New Control Method for Permanent Magnet Synchronous Machines with Observer". *35th IEEE Power Electronics Specialists Conference(PESC)*, 1024-1028.
- Cirincione M, Pucci M, Cirincione G,etal(2004). "A New Adaptive Integration Methodology for Estimating Flux in Induction Machine Drives". *IEEE Transactions on Industry Applications*, 19(1), 25-34.
- Shin-Myung Jung, Jin-Sik Park, Hag-Wone Kim, and Myung-Joong Youn (2008). "Improved Rotor Position Estimation employing Voltage Distortion Compensation for Sensorless PMSM Drives at Low Speed", *Power Electronics, 2007. ICPE '07. 7th International Conference on*, 2, 4243-4248.